of decimally-expressed integers, common fractions, and sexagesimals. Both are inferior to the masterly "Key to Arithmetic" of al-Kāshī (fl. 1400).

14. New Information on the Life of ʿAlī Qushchī, by A. Urunbaev (pp. 242-248).

The subject of this paper, who died in 1474, was an associate of Ulugh Beg at the Samarqand observatory. After the prince's assassination he left Central Asia, settling finally in Istanbul. His presence there may have had something to do with the reported Turkish use of decimal fractions at about that time. Here published are Russian translations of three notes (with facsimiles of the Persian originals of two) written by a friend of ʿAlī concerning issuance of road passes for the leg of his trip from Herat to Tabriz.

GIUSEPPE PEANO. By Hubert C. Kennedy. Basel (Birkhäuser Verlag). 1974. 31 p. (Beiheft 14 of *Elemente der Mathematik*)

Reviewed by A.F. Monna
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This is a monograph on the life and work of Giuseppe Peano. Until recently Peano's original work was not easily accessible. This changed after the publication of the *Opere Scelti di Giuseppe Peano* (Rome, Edizioni Cremonese, 1957-9), but the difficulty remained that Peano -- like most of the Italian mathematicians in this period (Volterra, Pincherle, Arzelà) -- wrote mostly in Italian. To help those who want to be acquainted with the original articles, the author of the present monograph has published English translations of selected papers (*Selected Works of Giuseppe Peano*, University of Toronto Press, 1973). This monograph on life and work is a welcome complement to these translations.

In describing Peano's life, from his youth to his development towards a primary interest in creating an auxiliary international language, the author gives many interesting facts, not only on Peano himself, but also on the circumstances under which he developed as a creative mathematician. It is interesting to learn that Peano had to study chemistry, zoology and drawing at the University of Turin and to compare his curriculum with ours. Kennedy gives details on the professors who influenced Peano. Among them Angelo Genocchi takes the first place (Genocchi began his career as a professor of Roman law and ended it as a professor of mathematics!). It would have been interesting to consider the general circumstances in those years against the background of the Risorgimento, through which Italy attained unity. Volterra,

for instance, who lived in the same period as Peano, was close to these movements (see the Preface of the Dover edition of Volterra's *Theory of Functionals and of Integro-Differential Equations*). Was there any relation between the Risorgimento and the high level of mathematics? However, such questions call for a broad study, perhaps beyond the scope of the monograph.

Peano is best known for his postulates of the natural numbers and his example of a space-filling curve, but he himself considered his work in analysis more important. He was famous for his capacity to find mistakes in the theorems of analysis; he was a man of counter-examples. In several fields he was a pioneer, for instance in the theory of measure, the axiom of choice, the introduction of the symbols now commonly used in set theory. He helped spread the ideas of H. Grassmann. Kennedy describes the shift of Peano's interest towards mathematical logic, and his editing of *Rivista di Matematica*, from which there issued the *Formulaire de Mathématiques*. There is information on Peano at the first Congress of Mathematicians (Zürich 1897) and the second (Paris 1900), where Peano inspired Russell's work on the foundations of mathematics. In those years Peano shifted his interest from mathematical logic towards promoting an international auxiliary language (*Latine sine flexione* or Latin without grammar); this is sketched in detail.

Some remarks of a more technical character:

The statement (p. 11) that "H. Lebesgue 1902 auf Anraten E. Borels vorschlug, an Stelle der in Peanos Definition verwendeten endlichen Menge von Mengen eine abzählbar unendliche Menge von Mengen zu setzen," is not quite correct in so far that Lebesgue did not come to his theory by following the advice of Borel. Lebesgue knew Borel's work but he came to his theory in his own way. The priority controversy between Borel and Lebesgue, lasting many years, is well known.

To the subjects in which Peano pioneered, the author could have added the system of axioms of a linear space. In his book of 1888, *Calcolo geometrico, secondo l'Ausdehnungslehre di H. Grassmann, preceduto dalle operazioni della logica deduttiva*, Peano gives in Chapter IX an almost modern definition of the concept of a linear space, formulating axioms that appeared again only many years later. He even considered infinite dimensional linear spaces and gave examples. Linear mappings are introduced in a coordinate-free way. Peano seems to have been the first to introduce these concepts in the modern abstract formulation, but without attracting the attention that they deserved.

The picture of Volterra as a representative of "die ältere Generation," with "konservativen, traditionsgebundenen Ansichten," seems too restricted. Volterra, with his "fonctions de lignes," was an important forerunner of functional analysis and his work in this field was highly original.

But these are only minor matters. The monograph is most

valuable because it shows that Peano's influence is greater and covers a broader field than seems to be known. The book gives more than a sketch of Peano's life and work. It contains interesting information on the general situation of mathematics at the turn of the century. It should be observed that the work of Italian mathematicians in the last decades of the nineteenth century -- at least their results in the field of analysis -- seems not always to be recognized as it ought to be. This calls for further research and publication. The present monograph is a first step.

The pamphlet contains a picture of Peano and a facsimile reproduction of a letter from Peano to Bertrand Russell.

GOETHE KAK ESTESTVOISPYTATEL [Goethe as a Natural Scientist]. By I.I. Kanaev. Leningrad (Izdatelstvo "Nauka"). 1970. 466 pp.

Reviewed by Martin Dyck
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"Goethe as a Natural Scientist" strikes one as a balanced summation of the poet-philosopher-scientist's activities and axioms in Goethean Science. Mr. Kanaev, an admirably Goethe-oriented *Vermittler von Objekt und Subjekt*, negotiates the pivots of Goethean Science on the elusive boundaries of the objective and subjective. Six chapters successively deal with Goethe's more immediate immersion into nature and its meaning ("Iz zhizni Goethe-naturalista"), with morphology, the theory of color, with "inorganic nature," and in the final chapter, with the intermerging of the poet's artistic and naturalistic endeavors ("Nauka i iskusstvo"). A 26-page chart ("Kratkaya sinkhronisticheskaya tablitsa") -- "*dia*synchronic" would be more accurate -- juxtaposes in six columns the major events of Goethe's life, his scientific pursuits, his poetical creations, all with concurrent developments in science and technology, in philosophy, literature, and the arts, and lastly, in the first column, with socio-political history. The chart reader's rising expectation drops to disenchantment in the text proper of the book: the author delicately desists from more explicitly correlating the first column with the fifth, or any other. Evidently the chart was meant merely to tantalize the reader's imagination (as a sort of appetizer to trick the reader to bring his own food).

A 19-page bibliography, inevitably selective, is conspicuous with certain non-selections: hardly from nescience, due perhaps to the ticklish ideological problem of grappling with such standard studies as Friedrich Gundolf's brilliant Goethe book of